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Can We Hear the Seabed Lithology from a Shipboard?

Abstract

The question from the title is a pragmatic paraphrase of the famous "Can one hear the shape of a drum?" [1]. We put it to emphasize that the acoustic information gathered with an ordinary hydrophone streamer is sufficient for depth elastic imaging the sea-bottom beneath, i.e. one can get depth sections of three medium parameters referring to elastic waves (such as, e.g., V_p/V_s-ratio, P-impedance and density, or their equivalents). In its turn the three images can be converted into the desired lithologic section. In this sense the question from the title has the positive answer: one does can get the depth lithologic image from hydrophone records directly.

Introduction

A new approach to the nonlinear waveform inversion - depth imaging of the seabed - is suggested. We show that the information recorded with hydrophones is sufficient for getting all the three of elastic parameters of the seabed with a fairly high resolution even in the case when "a near 1-D environment" is the only prior information. A string of our processing involves: a novel demultiple technique of seismic data which allows us to extract just primary events [2], a source wavelet estimation which provides us with reliable timing of primaries, and a new waveform inversion of full records ($\tau - p$ gather of raw data associated with a local lithologic structure). Jointly with *a priori* geological info and correlation links of bulk/shear moduli and density with lithologic parameters it can yield a lithologic section of the seabed beneath, provided the processing of seismic data is done properly.

We illustrate the seismic related part of our approach with 1.5-D synthetic data (in default of *a priori* info: as fewer bounces as possible are in use) as well as with field data from a Gulf of Mexico dataset, that was used in the SEG workshop on Multiple attenuation in 1998.

Method and applications

Our inversion scheme is based upon the new criterion on registered wavefields, which we called Ψ-criterion (Ψ stands for PSI : *p*artial *s*ynergetic *i*maging). It implies a direct nonlinear one-to-one mapping of $\tau - p$-transformed CMP's into ray parameter-depth section and allow us to avoid the traditional error-prone picking of layer/bounce-related events and any apriori parametrization of the seismic section. Most importantly, the Ψ-criterion makes the use of all the information contained in registered wavefields, not just that related to amplitude maxima. *P*artial *S*ynergetic *I*maging) means: • *p*artial- with respect to the varying ray parameter p and wavefield P-P, P-S, ... components - migrated depth images should be similar, when the related phase shifts are correct • correct $\mathbf{\Psi}$-gather (*z*-*p* domain) should be smooth enough along p and non-smooth with depth z • the result of the mapping (let us call it the $\Psi-$*gather*) preserves all the information jointly with the *correct* $P-$kinematics, which is dominant for the most of registered seismic events; thus rather rough density and $S-$velocity estimates give a "good" elastic medium model, which lies in a close vicinity of the global minimum of the wavefield misfit function. In its turn, such a model allows for applying of any scheme of linearized wavefield inversion (with respect to any full set of functions on bulk/shear moduli and density).

Ψ - criterion. Let us write the (unregularized) Ψ -criterion for nonlinear inversion in the form of the following optimization problem: *given* the $\tau - p$-gather $\mathbf{\Phi}$, *find* such a mapping $\mathbf{\Phi} \longrightarrow \mathbf{\Psi}$, *that*

$$\mathcal{J}_0 \;=\; \int \mathrm{d}x \,[(\,\mathbf{t}\cdot\nabla)\mathbf{\Psi}(x)]^2 = \int |\frac{\partial x}{\partial y}|\; \mathrm{d}y\,[(\,\tilde{\mathbf{t}}\cdot\nabla)\mathbf{\Phi}(y)]^2 \longrightarrow \min\;,$$

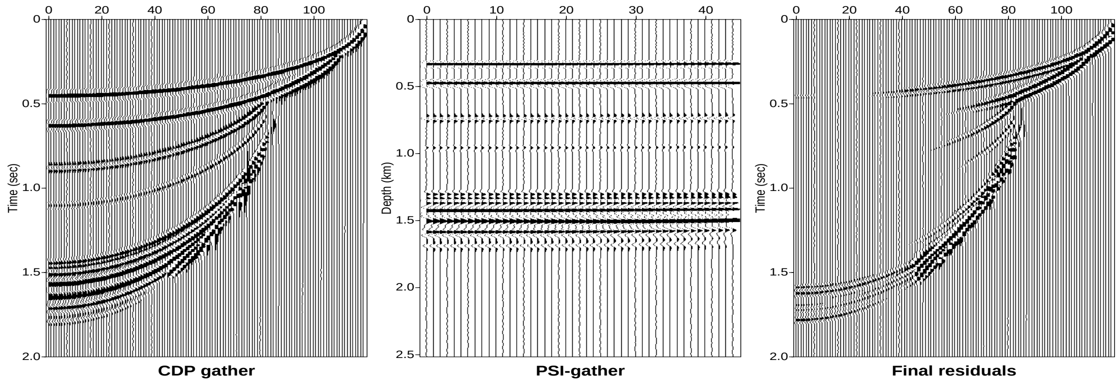

Figure 1: **Example of the inversion flow**: synthetic data (model in Fig. 2) • `Left:` input $\tau - p$ gather; • `center:` $\mathbf{\Psi}$-gather; • `right:` wavefield residuals after linearized inversion

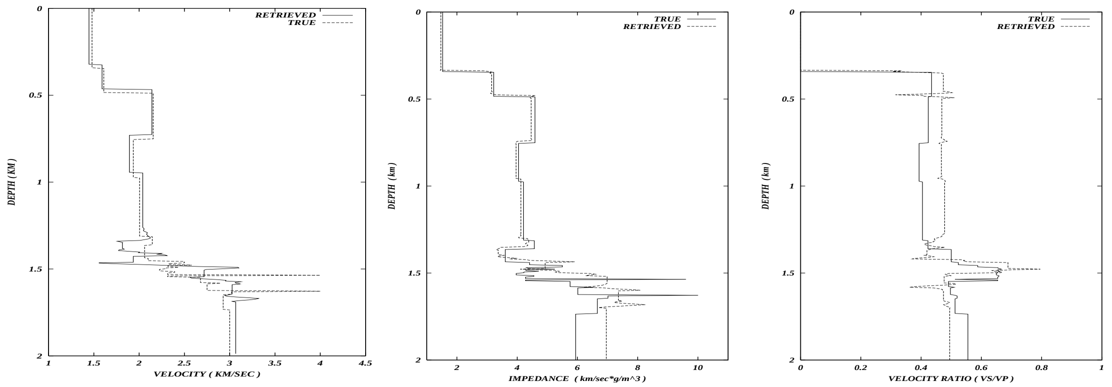

Figure 2: **Input model** and **result of full elastic inversion.** • `Left:` P-velocity; • `center:` P-impedance; • `right:` V_s/V_p-ratio.

with $\mathbf{\Psi}(x) = \psi(z,p)$ and $\mathbf{\Phi}(y) = \phi(\tau,p)$, the Jacobian $|\frac{\partial x}{\partial y}|$, the eikonal $\tau = \tau(z,p) = \int_0^z (s^2 - p^2)^{1/2} \mathrm{d}z$, while the varying with depth slowness $s = s(z)$ obeys to the eikonal equation: $(\mathbf{p},\mathbf{p}) = s^2$ with the ray vector $\mathbf{p} = \nabla\tau$ (an *italian* "p" stands for the horizontal component of the ray vector $\mathbf{p}$), and (*NB!*) the mapping $\mathbf{\Phi} \xrightarrow{\tau} \mathbf{\Psi}$ is governed by the eikonal $\tau_0 = \tau(z,0)$.
This optimization problem has an infinite set of extremals $\{\tau\}(\{s\})$, and to get a robust solution a proper regularization is needed, so the *regularized* $\mathbf{\Psi}$ - criterion is given with $\mathcal{J}(s) = \mathcal{J}_0(s) + \alpha\mathcal{R}(s) \longrightarrow \min$ where the regularizer $\mathcal{R}(s)$ is chosen as an energetic norm squared on s^{-1} (quadratic with respect to $v = s^{-1}$ form with the positively definite operator $\mathcal{H}$:

$$\mathcal{R}(s) \;=\; \| \, v - v_0 \, \|^2_{\mathcal{H}} \;=\; \int_0^{z_{\max}} \int_0^{z_{\max}} \mathrm{d}z' \, \mathrm{d}z'' \Delta v(z') H(z', z'') \Delta v(z''),$$

where $\Delta v = v - v_0$, and the operator $\mathcal{H}$ has an integral kernel $H(z', z) = H(z' - z)$ (spatial convolution) such that $\int_0^{z_{\max}} \mathrm{d}\tau_0(z) \, H(z' - z) \, \exp\{-|z - z''|/\Delta z\} = \delta(z' - z'')$, where $\mathrm{d}\tau_0(z) = s_0(z)\mathrm{d}z$. *Mathematically:* the (unbounded pseudo-differential) operator $\mathcal{H}$ in the quadratic form represents a modified Tikhonov's regularization (*actually:* the regularization claims that relative variations of p-velocity- in a scale, defined with Δz - should be smooth enough: see. e.g. [3]).
Fréchet derivatives $\mathcal{J}'_s$ can be written in terms of analytical functions on gradient components of the input $\tau - p$-gather $\mathbf{\Phi}$.

P-velocity. $\Psi-$criterion, governing the nonlinear isomorphic mapping of $(\tau - p)$-transformed CMP's into ray parameter-depth section $(z - p)$, produces an extra output: a phase-shift $\tau(z)$-curve, compatible with registered seismic events. *Example* is given in Figs.

Density. *Given:* p-partial images (wavefield depth extrapolation with the $\tau(z)$-curve/P-velocity) and source wavelet, *estimate:* density. *Modus:* When $p \to 0$, the effect of the shear modulus

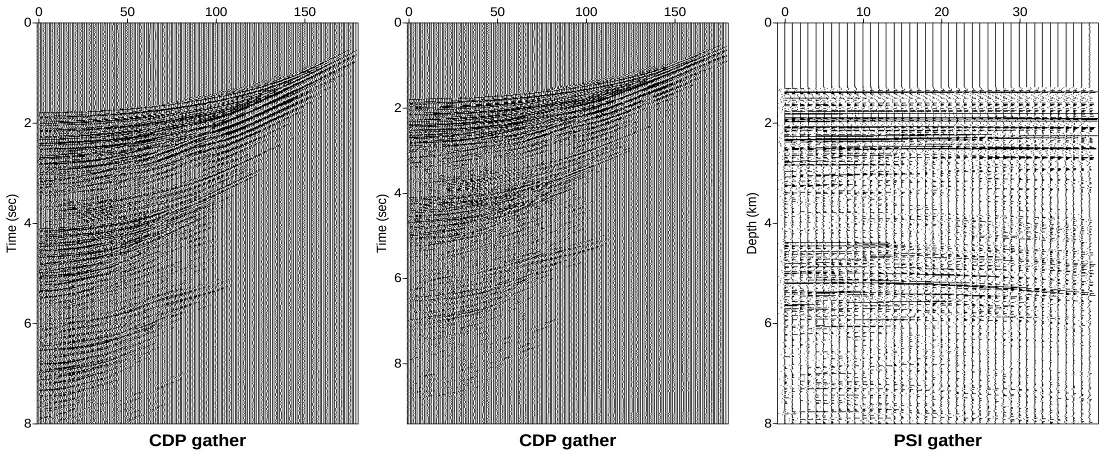

Figure 3: **Real data** from the Gulf of Mexico (deep see case). • `Left:` input $\tau - p$ gather; • `center:` demultiplied (via *SoSi-inversion* [2]) gather; • `right:` $\mathbf{\Psi}-$gather (via optimizing the $\mathbf{\Psi}-$criterion)

is negligible, hence variations of wavefield amplitudes correspond those of the pseudo-acoustic impedance; provided P-velocity variations are estimated, the related density variations are obtained with averaging over p-interval.

S-velocity. *Context:* given P-P events, with slight/no errors in the phase-shift (Ψ - criterion!), contribution of a shear modulus is weak: under these conditions the *estimate* of the S-velocity V_s can be rather rough: e.g., from $V_p = \sqrt{3}V_s$ (hence V_p/V_s-ratio is constant in such a background model, see however the result of the linearized elastic inversion: Fig.2, `right`).

This strategy allows for revealing a fairly good background model with no trace of nonlinear effects (no phase shifts) and helps us to get for sure into the vicinity of the global minimum of the conventional misfit function for elastic inversion (the function on residuals of observed/synthetic wavefields). It results in essentially higher spatial resolution, than that for conventional approaches to processing such data.

Linearized waveform inversion. The linearized waveform inversion is performed via traditional objective function: residuals of real and synthetic wavefields, while the corresponding Fréchet derivatives are evaluated by numerical differentiation of the reflectivity codes under the given ("good") background model.
Unlikely to conventional approaches we (can) use the raw data (multiples and converted waves are included) as the counterpart of the corresponding synthetic wavefields in the misfit function.

Discussion

We omitted intentionally the use of any apriori information in the "seismic related" part of the approach. The assimilation of such an (independent, but expensive) information can be done properly, as long as estimated through the linearized inversion elastic parameters have as an attribute correlation functions of their uncertainties.
At last the "elastic" image of the seabed can be converted into the geological section, provided geological information about the the field and related correlation links of bulk/shear moduli and density with lithologic parameters are given.

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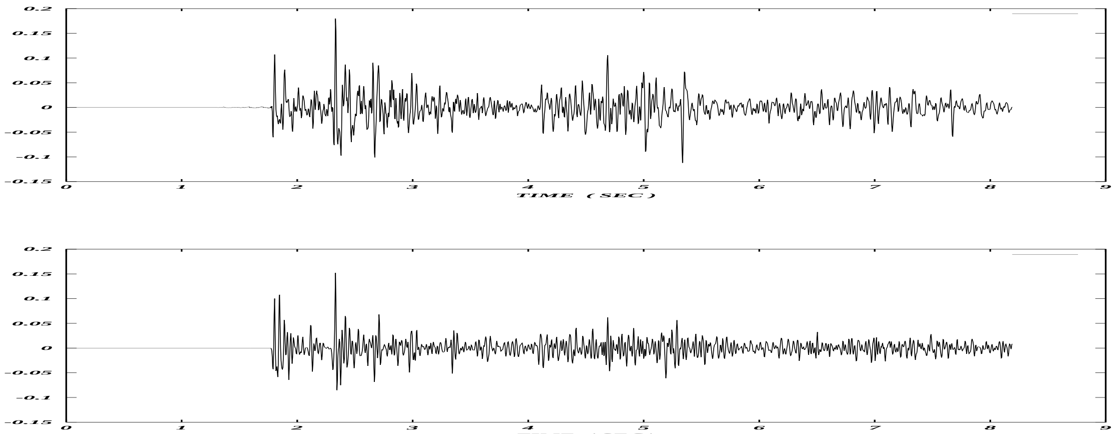

Figure 4: **Comparison of real and synthetic traces.** • `Top:` real trace from the $\tau - p$ gather (Fig. 3, `left`, the "near-0-p" trace) • `bottom:` the corresponding synthetic trace, generated by the reflectivity routine with the elastic parameters revealed by the approach (Fig. 5). The correlation coefficient for these two traces is 0.45, the averaged one - over the first 50 $p-$ traces- is 0.35.

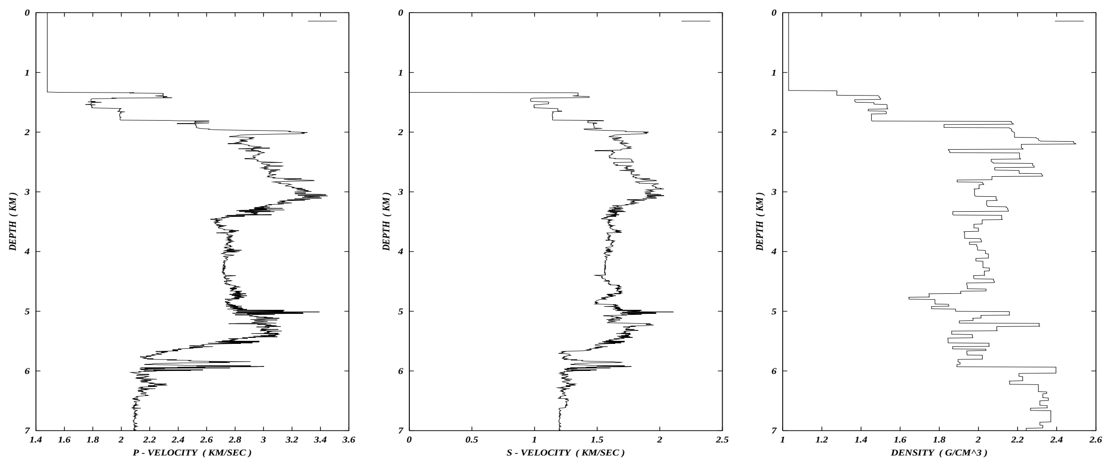

Figure 5: **Result of full elastic inversion**: real data (in Fig. 3). • `Left:` P-velocity; • `center:` S-velocity;; • `right:` density.

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