# Removal of intrabed multiples via Source-Signature Invariant Inversion

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## Summary

A novel approach to comprehensive removal of multiples,- including intrabed ones, - is suggested: **Inverse Scattering**-strategy is merged with that of **Predictive Deconvolution**. The method is based upon a Source-Signature Invariant Inversion (hereafter **SoSI-inversion**), i.e. such an inversion of reflection data with respect to unknown reflectivity, which is not dependent on a source timing/wavelet. Recursive form of the inversion gives sequential elimination of multiples: the first step is responsible for elimination of free-surface related multiples, while a total number of steps is induced by related number of strong reflectors, causing intrabed reverberations.

## Introduction

The problem of removal of multiples is tied to the rather sophisticated problem of nonlinear inversion: Strategies of demultiplying data from essentially 3D-environment are based upon the **Inverse Scattering**- strategy (e.g. (Weglein and Matson, 1998), (Berkhout, 1999)) but relevant algorithms are still under development, besides it is expectable they could be computationally expensive.

Classical approaches to suppression of multiples in a locally 1D-environment exploit a well-seen self-repetition of records (Robinson and Treitel, 1980). In practice it concerns mainly the multiples generated by the strongest reflector, i.e. surface-related ones. In the most of applications it gives fairly good results, but it is not so good, when, for example, a sea-bed contains a long sequence of rather strong reflectors or a deep sea-floor is formed by a thin-layer slab with basalt beneath: the latter yields intrabed reverberations embedded into water-layer ones.

The suggested SoSI-strategy for elimination of multiples utilizes, in fact, the existence of multiples themselves: due to reverberations the initial segments of registered wavefields can be considered as a long-acting source of fairly sophisticated signature thus providing us with a well detectable media parameters under uncertainties in an actual source timing/location/signature. Posing the problem as a 3-D inverse scattering one, we arrive to the strategy of predictive deconvolution when the problem can be reduced to 1.5-D, thus implying a way to substituting computationally expensive approaches to processing data from complex "3-D" geological provinces, with more robust and fast ones.

## SoSI-inversion & Elimination of multiples

An initial trick consists of assumption that we can represent a stratified medium with a (still unknown) kinematically correct but nonreflecting half-space, assuming that we always can substitute bounces with transition zones. It allows us write the Lippmann-Schwinger equation formally as

$$\varphi \;=\; \varphi_{\downarrow} \;+\; \mathbf{G}\;\mathbf{V}\;\varphi \qquad (1)$$

where $\varphi_{\downarrow}$ is *downward* wavefield in a reference media that is supposed to include free-surface boundary conditions, but to be a smooth approximation of a true impedance section (no reflections: the concept that we exploited in (Ryzhikov et al., 1995) to solve a 3-D nonlinear problem for reflection data). $\mathbf{G}$ is the corresponding Green operator and $\mathbf{V}$ is the perturbation, induced by the difference between the smooth model and true section with bounces. The advantage is that we can deal now with spatially separated potentials, order the latter with respect, e.g., a downward direction, so that index $i$ allows us to count/nominate multiples, and so forth: $\mathbf{V} = \sum \mathbf{v}_i$, where the closest to the free surface bounce has index 1, and we keep the index 0 for the free surface.

To get a data $d$ recorded on a registration surface, we apply operator $\mathbf{\Gamma} : \mathbf{\Gamma} \;=\; \mathbf{\Gamma}^{\dagger} \;=\; \mathbf{\Gamma}^2$ which "extracts" data $d \;=\; \mathbf{\Gamma}\varphi$ - values of the wavefield $\varphi$ on the registration surface, and thus

$$d = \mathbf{\Gamma}\mathbf{G}(\sum \mathbf{v}_i)\varphi = \sum p_i + \mathbf{\Gamma}\mathbf{G}(\sum \mathbf{v}_i)\mathbf{G}(\sum \mathbf{v}_j)\varphi \quad (2)$$

where we omit $\varphi_{\downarrow}$ (it does not contribute into registered $d$), $\sum p_i$ represents a sum of **desirable primaries**, induced by the first item of the conventional Born series, but the equation still contains unknown $\varphi$ in every point of the medium. In the chosen reference medium the Green operator $\mathbf{G}$ is a one-way operator, composed with two parts because of the *half*-space (besides it counts transmition losses), and as long as we are dealing with *reflection* data, every possible turning point can be marked with an index $i$ of relevant interface/potential $\mathbf{v}_i$.

Substituting $\mathbf{G}\mathbf{v}_i \;\equiv\; \mathbf{r}_i$, so that $\mathbf{r}_i$ is the one-way propagator of reflected on a $i-$bounce wavefield, e.g, $\mathbf{\Gamma}\mathbf{r}_i$ represents propagation from the $i-$bounce to the registration surface and includes corresponding traveltime shift, we rewrite the equation 2 in a form

$$d \;=\; \mathbf{\Gamma}(\sum \mathbf{r}_{i'})(\sum \mathbf{r}_{i''})\; \overset{\leftrightarrow}{d} \;+\; \sum p_i \qquad (3)$$

where $\overset{\leftrightarrow}{d} = \mathbf{G}^{\dagger}\mathbf{\Gamma}\varphi$ is a backward propagated data [1]. Equation 3 is a closed form for SoSI-inversion: formally there is no source function there.

Elimination of multiples is associated with the following recursive representation of the equation 2:

[1] note, that *multiples* in equation 3 corresponds to *broken ordering* of indexes in products $\mathbf{r}_{i'}\mathbf{r}_{i''}$

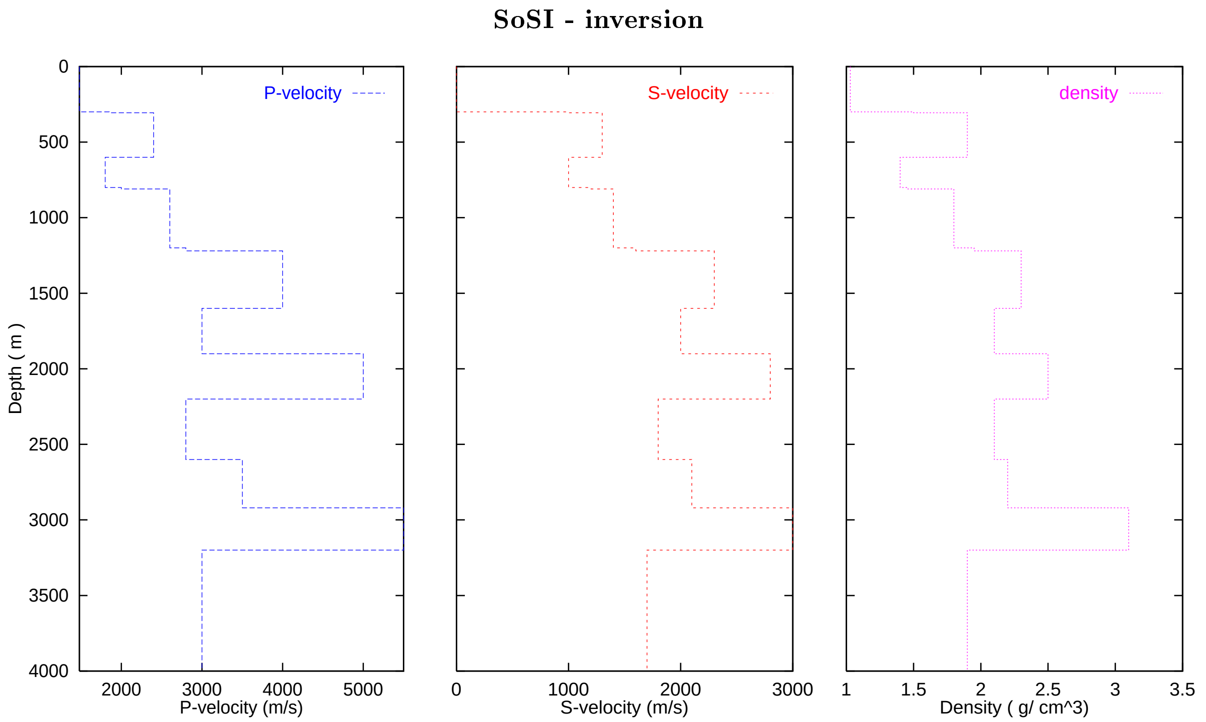


Fig. 1: Model used for simulating blind 1.5 D marine seismic data: P-velocity; S-velocity; density.

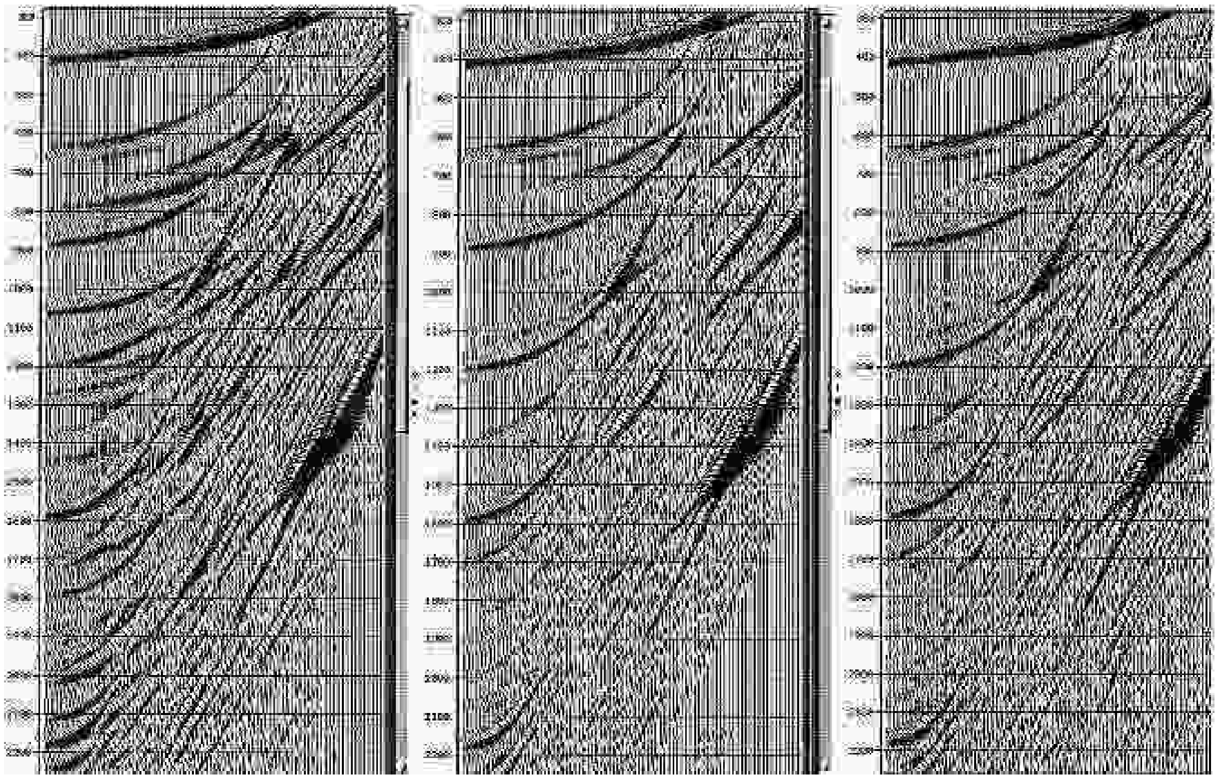

Fig. 2: **SDec** (Sharp Decon: (Ryzhikov and Biryulina, 1998)) **in p - tau domain.** From left to right: **1. input** (`left`) : blind data in p-tau (CSG for the model from Figure 1 - we got the model a posteriori); given: the shortest offset = 100 m, offset increment = 25m, source depth = 6 m, geophones' depths = 6 m; no information on source signature); **1. 2. theoretical output** (no induced by free surface multiples); *in other words, this gather represents the very best result of elimination of surface-related multiples* **3. SDec - output.** Note, that it is not a final solution: records contain still lots of multiples to be removed (compare **2** and **4** Figure 3).

## SoSI - inversion

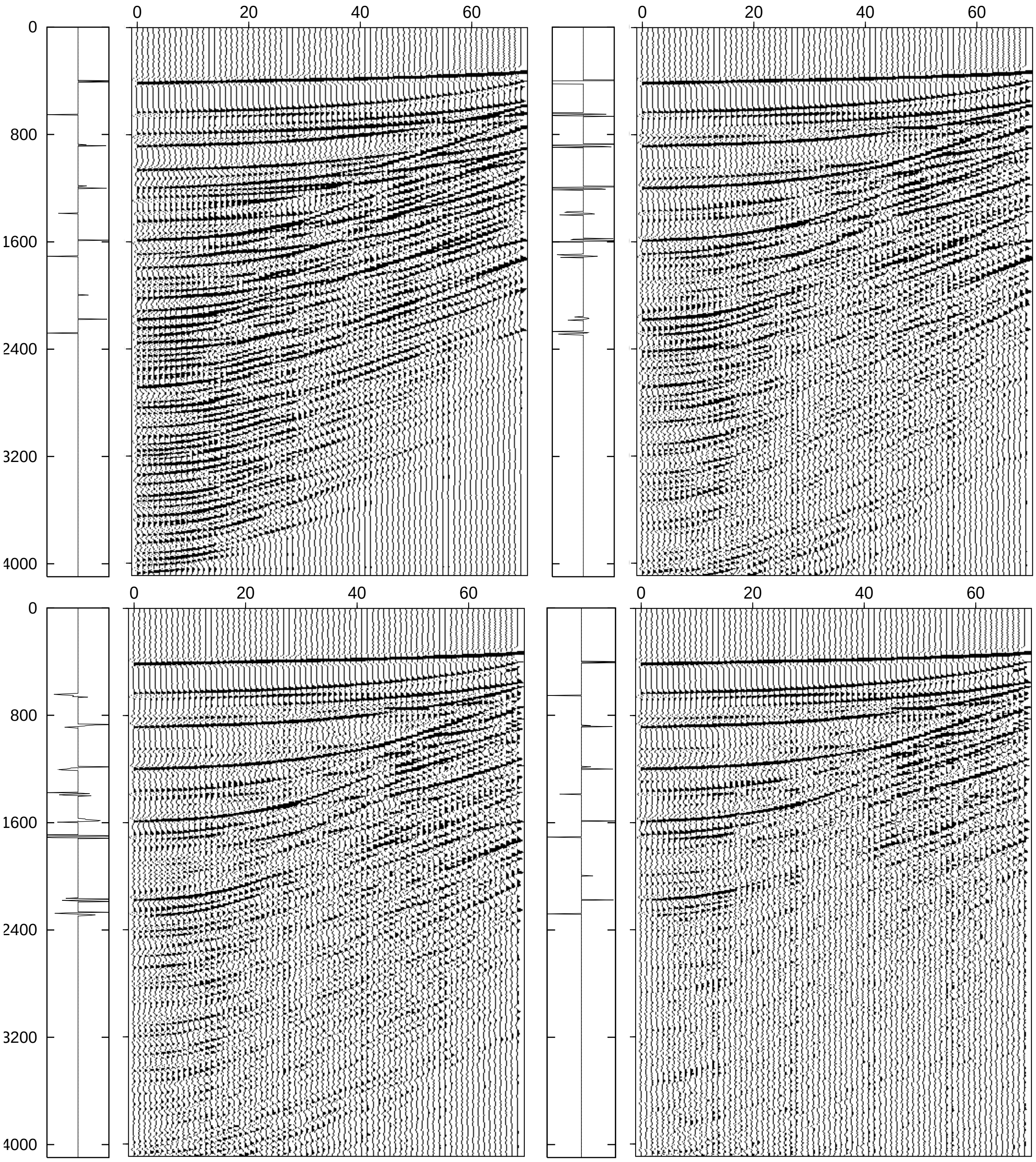


Fig. 3: **SoSI-inversion:** example of **intrabed multiple removal** from synthetic 1.5 D marine seismic data (related model is given with Figure 1) `TOP` • **1. input: CMP-gather in $p-\tau$ domain**; • **2. output of the 1-st iteration:** free-surface related mutiples are removed (comparison with the respective theoretical output is given with Figure 2) `BOTTOM` • **3. output after the 2nd step of recursion:** the first portion of peg-legs and intrabed multiples associated with the sea-floor is removed; • **4. the final output** after the 10th step. `Narrow plots nearby:` • Top: **1. theoretical impulse trace** ($p = 0$ : normal incidence) induced by the model; **2.** prediction-error **filter coefficients of the 1-st step** of recursion; • Bottom: **3. filter coefficients of the 2-nd step 4.** again: **theoretical impulse trace.**

$$d_j \;=\; [\,\mathrm{I} \;+\; \boldsymbol{\Gamma}(\sum_{i=j+1} \mathbf{r}_i)\mathbf{r}_j]^{-1}\,(\,d_{j+1} \;+\; p_{j+1}) \qquad (4)$$

**1.5-D data**

Recalling the eikonal equation $(\mathbf{p},\mathbf{p}) = s^2$ with the slowness vector $\mathbf{p}$ and an inverse velocity squared $s^2 = s^2(z)$, depending just on depth $z$, let us interpret equation 4 by the following way:

• as far as travel time is $\tau(z) = \int_0^z \mathrm{d}z\,(s^2(z) - p_x^2)^{1/2}$, where $p_x$ is a parameter of horizontal slowness in the $p-\tau$-domain, equation 4 can be written in time-/$\tau-$domain.

• Green function $\mathbf{G}_0(z', t; z, 0) \propto \delta(t - \Delta\tau)$ with $\Delta\tau = \int_z^{z'} \mathrm{d}z\,(s^2(z) - p_x^2)^{1/2}$, which is travel time from $z$ to $z'$;

• $V$-"potential", e.g., for the wave equation, when $p_x = 0$ $V(z) \propto \mathrm{d}(s^2(z))$., while

• Jacobian for integrating Eq. 4 is $\frac{\mathrm{d}}{\mathrm{d}z}\tau = (s^2(z) - p_x^2)^{-1/2}$

The 1st step of recursion 4 is read now as

$$d_0 \;=\; [\,\mathrm{I} \;+\; \boldsymbol{\Gamma}(\sum_{i=j+1} \tilde{\mathbf{r}}_i)]^{-1}(\,d_1 \;+\; p_1\,) \qquad (5)$$

where $\tilde{\mathbf{r}}(\tau, p)$ stands for the *effective reflectivity* and involves doubled travel time from the free surface to relevant interface.

Now one can see easily the transition *"Inverse Scattering"* $\Rightarrow$ *"Predictive Deconvolution"* to be accomplished: linearization $d_1 \to d_0$ gives a schema of predictive deconvolution, which is based upon the assumption that peg-legs and intrabed multiples are negligible in $d_1$. An example when this assumption fails is given with Figures 2 and 3.

To get reliable estimates of reflection coefficients (reverberation operator) we put in action SDec (Sharp deconvolution: (Ryzhikov and Biryulina, 1998)). An example of SDec-run is presented by Figure 2.

## Conclusion and discussion

Every step of SoSI-recursion should yield resonable estimates of reflectivity, as long as all of intrabed multiples are generated by the same reflectors (see, e.g., Figure 3). Therefore the method provides analysts with a tool for a quality control based on self-consistency of sequential elimination of multiples: it is well-known that the criterion of "minimal energy" in predictive deconvolution can lead to unreasonable errors while processing high-quality data.

The use of the method in essentially 3-D environment needs in fast migration of CSG-data jointly with estimation od a relevant kinematic model (see, e.g,, a relevant strategy revealed by the the authors earlier in (Ryzhikov et al., 1995)). The "locally 1-D"-approximation, when applicable, allows one to reduce the strategy to that of predictive deconvolution and thus to get very fast and robust algorithms, capable of running in a fully automatic mode. Hopefully, after an extensive training with different sets of real data the method can become a reliable part of routine processing. [2]

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[2] Note, that for processing 3-D data, corrupted by incoherent multiples the strategy of SoSI-inversion/SDec admits an extension (Biryulina and Ryzhikov, 1999) - not in terms of waveforms but so called energetic envelopes. To process 100 x 2048 traces takes a few seconds on a HP PC.