Convex Body Regularization (CoBR) in linear inversion/deconvolution

Marina Biryulina and Gennady Ryzhikov

University of Bergen, Institute of Solid Earth Physics, Allégt.41, 5007 Bergen, Norway;
E-mail: Ryzhikov.Gennady@uib.no

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Introduction

Conventional L_2-inversion as well as statistical estimation do not allow us to detect inconsistency in a 'forward model' ⇔ 'estimates of noise' ⇔ 'real data'. It happens because a misfit function tends to null provided the number of parameters under estimation is large enough. A priori information given in terms of covariance or smoothing operators does not help much if not yields a strongly biased solution. An alternative approach is suggested which combines linear and quadratic programming: a priori information is approximated by constrains on values of a set of linear functionals on a reconstructed function which does not exclude initial quadratic constrains/regularization. In general a linear functional set induces a very complicated *convex body* in an initial functional space that prevents an applying of iterative or random searching algorithms. The suggested *Convex Body Regularization* inversion algorithm, or **CoBR**), is based on projection of an optimal L_2-solution on a set of 'physically admissible functions' in a space with a metric induced with generalized Fisher's operator. It leads to an 'embedding of projection problems' in the sense of consequential reducing of corresponding functional/Lagrange multiplier space dimensions and reducting of respective metrics. The CoBR-approach is illustrated by applying to real CDP-data.

Method and application

Let us formulate the CoBR-approach in terms of CDP-stacked data processing. The inverse problem is as follows: given the CDP-stacked data d_{obs} and a source wavelet $w(t)$, find an acoustic impedance $r(t)$ (hereafter we use the name *acoustic impedance* for *logarithm of acoustic impedance*) that minimizes the objective function $\mathcal{F}$, i.e. find such a $r^*(t)$ that is the argument of the minimum of $\mathcal{F}$:

$$r^* = \arg \inf_{r \in \mathcal{R} \subset \Re} \mathcal{F}(r) \ , \tag{1}$$

The model set $\mathcal{R}$ has to be chosen properly, and $\mathcal{F}^r$ denotes the function expressed in terms of observed- and synthetic- data residuals while a priori information is taken in terms of quadratic forms on $\{r\}$:

$$\mathcal{F} \ = \ \langle \ d_{obs} - d_{cal}(r) \ | \ d_{obs} - d_{cal}(r) \rangle \ + \ \langle \ r_{cal} \ | \ \alpha_1 \mathrm{I} - \alpha_2 \partial_t^2 \ | \ r_{obs} \rangle \ , \tag{2}$$

$d_{cal}(r)$ is the synthetic seismogram for an acoustic impedance $r(t)$ and d_{obs} is the CDP-data, and α_1 and α_2 are positive parameters of regularization. Additional regularization is introduced by subset $\mathcal{R} \subset \Re$ for the optimization problem (eq.1):

$$\mathcal{R} = \{ r \mid \{ b_n^{min} \le l_n(r) \le b_n^{max}, \ n = 1, 2 \ldots N \} \} \tag{3}$$

where l_n is a set of *linear* functionals, e.g. local/global values of impedance itself or its derivatives/'jumps'. Interval values of impedance or trend of it can serve as an example of a 'global' constrain. The constrains (eq. 3) define the *convex body* $\mathcal{R}$: It is easy to check, that the objective function (eq.2) can be rewritten as follows:

$$F \ = \ \langle \ r - \tilde{r} \ | \ \mathcal{H} \ | \ r - \tilde{r} \rangle \ \stackrel{\text{def}}{=} \ \| r - \tilde{r} \|^2 \tag{4}$$

where $\mathcal{H}$ is a positive regularized Fisher's operator, and $\tilde{r}$ is a solution of the optimization problem (eq.1) on the entire set $\Re$. Therefore the problem (eq.1) can be formulated as a problem of projection, in a space with a metric induced by $\mathcal{H}$ operator, of the solution $\tilde{r}$ on a convex body $\mathcal{R}$:

$$r^* = \arg \inf_{r \in \mathcal{R} \subset \Re} \| r - \tilde{r} \|^2 \ , \tag{5}$$

The CoBR-algorithm is constructed in terms of (positive) Lagrange parameters, the dimensions of the corresponding parameter spaces is induced with a number of properly chosen *active* constrains.

The input data for CoBR-inversion/deconvolution is following: 53 CDP-traces from a line profile (Barentz Sea area) with 25 m distance between points of the stacking (**Fig.1**, it CDP-gather) + source wavelet with covariance of error estimates. A corresponding forward model is a convolution of the wavelet original and impedance. The number of recovered parameters (acoustic impedance) is equal to the number of seismogram samples with 0.002 sec rate.

Linear constrains have been chosen as local: a variance of reflection coefficient for 0.002-sec events is less than 0.4; and 'global' ones: a restriction for an interval sum of absolute values of the reflection coefficients. Note: there is no constrain on a value of the acoustic impedance itself.

The results of inversion are displayed below (**Fig.1**, *Impedance*, *Reflectivity*), where we plot the reconstructed functions in a similar to CDP-data mode. To make an analysis of the inversion result more straightforward we use an additional well log data. The well location is associated with distance 50 m (**Fig.1**). On **Fig.2**, *top*, a thin curve of large variance represents the CDP-trace at the same time window, while the reconstructed trace as well as synthetic one generated by well log data are near the null level. This indicates an inconsistency in CDP-data and the source wavelet used for deconvolution. Nevertheless the reconstructed impedance has still much larger variance than well-log's one (thin and thick curves on **Fig.2**,*bottom* respectively).

Conclusion

CoBR-algorithm is fairly robust: the reconstructed horizons are smooth enough despite of 1D -inversion (no lateral smoothing has been introduced a priori) (**Fig.1**, *Impedance*, *Reflectivity*).

In contrary to conventional approachs CoBR-algorithm allows us to find the best 'physical' solution (e.g. to get a 'causal' filter); it provides an opportunity to filter very strong noise of unknown statistics, or to find an inconsistency in a forward model when a posteriori estimate of noise is far away from a confidence interval (**Fig.2**, *top*).

In contrary to global optimization algorithms (e.g. RGA-algorithm, [1]) CoBR-algorithm enables to get an optimum solution for huge number of unknown parameters; besides it gives the best estimate even in a case of an empty set of 'physical' parameters (e.g. when constrains are chosen incorrectly).

Acknowledgments

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References.

[1] Ryzhikov, G.A., Biryulina, M.S. and Keydar, S., 1996, Analysis of 1D seismic waveform inversion by Regularized Global Approximation: *Journal of Seismic Exploration, v.5,* pp.349-362

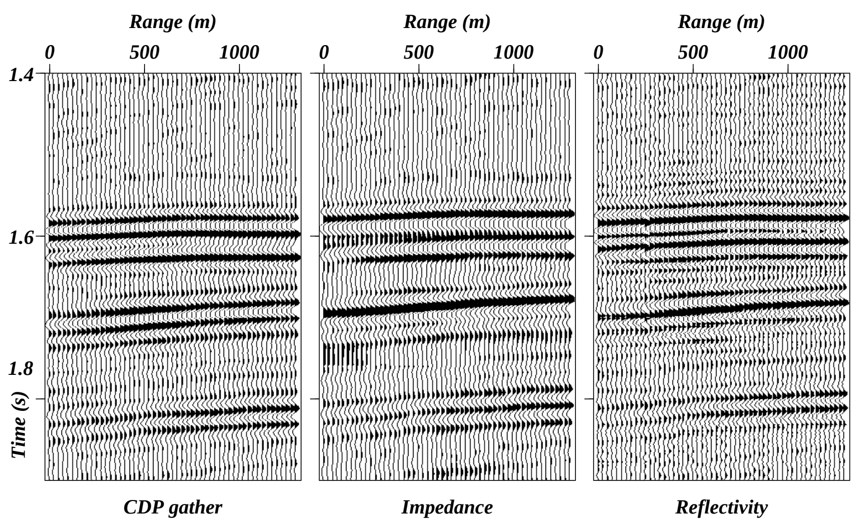

Fig.1. *Input CDP-data and 1-D inversion results.*

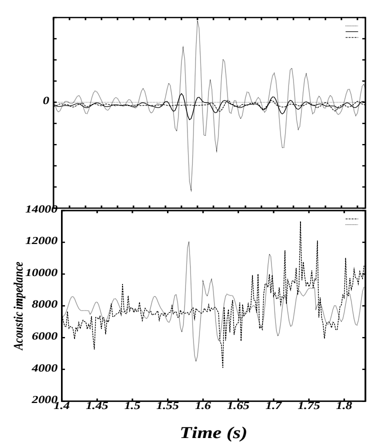

Fig.2. *Comparison with well log.*