METHOD OF EFFECTIVE SPECULAR POINTS

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Introduction

An approach for reconstruction of an interface geometry ("Reflector Geometry Reconstruction", or *RGR*) is suggested. It is supposed that the interface of interest is situated in 3D layered media, an interval velocity of the over-interface layer is under estimation also, while velocities in all other upper layers are assumed to be known precisely enough to define a reference kinematical model. The RGR deals with unstacked reflection data from a rather sparse and irregular in general source-receiver net $\{\mathbf{x}_s, \mathbf{x}_r\}$. The RGR treats the data as caused by *effective specular points*, or *e.s.p.*. It means that seismogram is supposed to contain such a fragment that can be interpretated as a result of reflection of a pulse with a single elementary 'mirror'. After a proper preprocessing the input data for the approach are signal traveling times estimated for all source-receiver pairs, t_{obs}, which should be interpreted as corresponding to traveling along a 'source-e.s.p.-receiver' ray $t_{cal}(\mathbf{x}_s; \mathbf{x}^*; \mathbf{x}_r)$. No parametrization of the interface is used, it is just supposed a priori that the unknown interface is rather smooth. The problem is to find all effective specular points (i.e. to find coordinates of the points and corresponding normal vectors), to define the velocity and to reconstruct the interface that should be tangent to all elementary 'mirrors' (e.s.p.'s).

Method and application

Let a set of travel times contribute the data set vector $\mathbf{T}_{obs} = \{t_i,\ i = 1, \ldots, I\}$, where index i corresponds to a source-receiver pair, and a symbol $\mathbf{X}^*$ stands for the set of e.s.p. locations: $\mathbf{X}^* = \{\mathbf{x}_i^*,\ i = 1, \ldots, I\}$. We assume that the interface φ is fairly smooth *a priori*, and its normal vector is oriented into the lower hemisphere, i.e. φ can be represented as a function of two variables: $\varphi = \varphi(x)$, where x denotes lateral coordinates ($\mathbf{x} = \{\ x,\ x_3\} = \{x_1, x_2, x_3\}$ and $\mathbf{X} = (X, X_3)$). The problem is posed as an optimization one:

$$(\hat{V},\ \hat{\mathbf{X}}^*,\ \hat{\mathbf{N}},\ \hat{\varphi}) \ = \ \arg \inf_{V, \mathbf{X}^*, \mathbf{N}, \varphi} \mathcal{F} \qquad (A)$$

The functional $\mathcal{F}$ involves four components:

$$\mathcal{F} = F_1 + F_2 + F_3 + F_4 = ||\mathbf{T}_{obs} - \mathbf{T}_{cal}||_1^2 + ||\mathbf{N} - \tilde{\mathbf{N}}||_2^2 + ||\varphi||_3^2 + ||V - V_0||_4^2 \qquad (A.0)$$

The first functional in eq.A.0 connects observed travel times $\mathbf{T}_{obs} = \{t_i,\ i = 1, \ldots, I\}$ and travel times calculated for the reference kinematics model plus one-parameter velocity layer: $\mathbf{T}_{cal} = \mathbf{T}_{cal}(\mathbf{X}^*) = \{\tau_i(\mathbf{x}) = \tau_i[\varphi(\mathbf{X}^*)],\ i = 1, \ldots, I\}$ denote a set of V-parametrical isochrones $\tau_i(\mathbf{X};\ V) = t_i + \varepsilon_i$. The error ε_i of an i-th travel time detecting induces the functional F_1 in eq. A.0:

$$||\mathbf{T}_{obs} - \mathbf{T}_{cal}||_1^2 = F_1[\varphi(\mathbf{X}^*); V] = \ \alpha_1 \sum_{i=1}^{I} C_{ij}^{-1}(t_i - \tau_i(\mathbf{x}^*; V))(t_j - \tau_j(\mathbf{x}^*; V)) \qquad (A.1)$$

where C_{ij} is the covariance matrix of errors $\{\varepsilon_i\}$. The second term in (A.0) includes $\mathbf{N}$, a "normal vector to an effective specular point", which denotes the set of vectors $\mathbf{n}_i$, normal to the corresponding V-parametrical i-th isochrone in the point with coordinates of i-th e.s.p.: $\mathbf{N} = \mathbf{N}(\mathbf{X}^*) = \{\mathbf{n}_i,\ i = 1, \ldots, I\}$ where $\mathbf{n}_i = \nabla_{\mathbf{x}_i^*} \tau_i(\mathbf{x}) / |\nabla_{\mathbf{x}_i^*} \tau_i(\mathbf{x})|$, and $\tilde{\mathbf{N}} = \tilde{\mathbf{N}}(\varphi(X^*)) = \{\tilde{\mathbf{n}}_i,\ i = 1, \ldots, I\}$ denotes the set of vectors $\tilde{\mathbf{n}}_i$ which are normal to the reconstructed interface φ: $\tilde{\mathbf{n}}_i \ = \ k\ (\ 1,\ -\nabla_{x_i^*} \varphi(x))$ with $k \ = \ (\ 1 + (\nabla\varphi\ , \nabla\varphi))^{1/2}$. The secomd squared norm in eq.A.0 is defined by an inner product in R^3:

$$||\mathbf{N}||_2^2 \ = \ \alpha_2\ (\mathbf{N},\ \mathbf{N}) \qquad (A.2)$$

This term can be replaced by $(\ -2\alpha_2 \sum_i \cos\ \widehat{\mathbf{n}_i \tilde{\mathbf{n}}_i})$. The third functional in eq.A.0 is regularizing functional, that implies the interface φ as a function of X belongs to a ball in *Sobolev space of infinite order* [1]:

$$||\ \varphi\ ||_3^2 \ = \ \alpha_3\ \langle\ \varphi\ |\ \ e^{\ -\beta \nabla^2}\ |\ \varphi\ \rangle \qquad (A.3)$$

where α_3 and β are positive parameters of regularization and scalar product in $\mathcal{L}^2$ is given by $\langle\ \varphi\ |\ \psi\ \rangle \ = \int\ dx\ \varphi(x)\ \psi(x)$. At last the forth functional is proportional to the squared deviation of a velocity to be estimated from a reference one:

$$||\ V - V_0\ ||_4^2 \ = \ \alpha_4\ (V - V_0)^2 \qquad (A.4)$$

The method of optimization in a certain sense is constructed as a kind of 'coordinate' descent: varying the weight factors α (eq.A.1-A.4), one can get reduced optimization problems, that can be solved much easier than the main problem (eq.A). The optimization scheme chosen by the authors is following:

$$[\alpha_2 = 0]_0 \Longrightarrow [\alpha_1 = \alpha_3 = 0]_1 \Longrightarrow [\alpha_2 = 0]_2 \Longrightarrow [\ldots]_1 \Longrightarrow [\ldots]_2 \ldots \qquad (B)$$

where the 0-th step $[\ldots]_0$ allows to get the 0-th order approximation for the interface φ, the velocity V and the vector e.s.p. $\mathbf{X}^*$ (neglecting the Snell's law), while the iteration process "$[\ldots]_1 \Longrightarrow [\ldots]_2 \Longrightarrow [\ldots]_1 \Longrightarrow [\ldots]_2 \Longrightarrow \ldots$" allows to specify the e.s.p.- set and the interface plus the velocity correspondingly.
The RGR-approach is illustrated with a simplified synthetic experiment (Fig.3a): the nearest, with respect to the source-receiver regular net, interface in a homogeneous media is of interest. To simulate a real experiment the input data was taken with noise $\varepsilon : E(\varepsilon/t_{obs})^{1/2} = 0.02$. The nice behavior of a reduced objective function (the 0-th step of optimization (B)) is shown with Fig.1 The final result of the interface reconstruction (Fig.3b) jointly with relative errors (Fig.2) indicates the applicability of suggested approach: the relative errors are of the same order as the relative noise in input data.

Conclusion

Under the assumption that the main part of reflection data is contributed with a 2D-reflector, the RGR-approach, or the *method of effective specular points*:
(1) is preferable than the ideology of linearized 3D-inversion: the latter does not involve a 'cooperative' effect of a reflector, besides it deals with a 3D-image and needs in consequential 'extracting' of a 2D-interface [2].
(2) allows to deal with essentially 3D inverse problem, when any a priori parametrization of a reflector is very restrictive. As a result, the RGR supplies the higher accuracy compared with conventional approachs.

Acknowledgments

The research was conducted under the EU project "3D Asymptotic Seismic Imaging" (Contract JOU2-93-0321) and partly under the AFOSR Grant # 49620-94-1-02878.

References.

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[2] Ryzhikov, G.A., and Biryulina, M.S., 1995, 3D nonlinear inversion by entropy of image contrast optimization: *Nonlinear Processes in Geophysics, v.2, no 3/4*, pp. 228-240

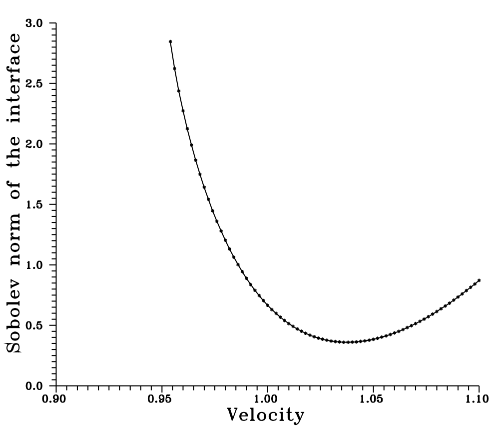

Fig. 1. NORM VALUE VERSUS RELATIVE VELOCITY

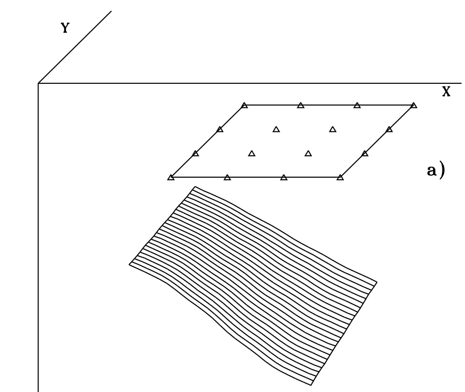

Fig.3a. GEOMETRY OF THE EXPERIMENT

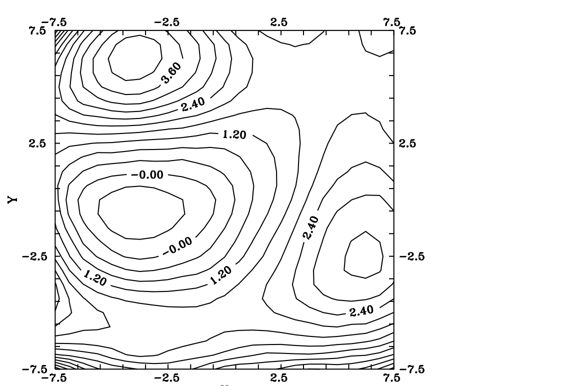

Fig. 2. RELATIVE ERRORS (per cent) OF THE DEPTH RECOVERING

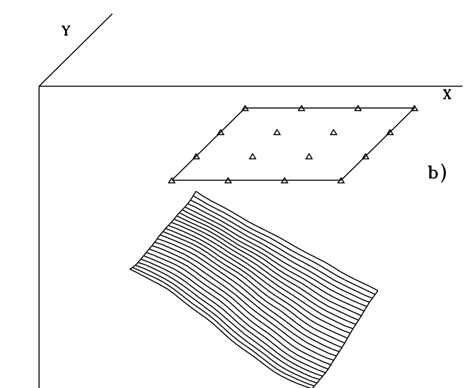

Fig.3b. RECONSTRUCTION OF THE INTERFACE